siam 2022 | ACM-SIAM Symposium on Discrete Algorithms

Optimal Oblivious Parallel RAM

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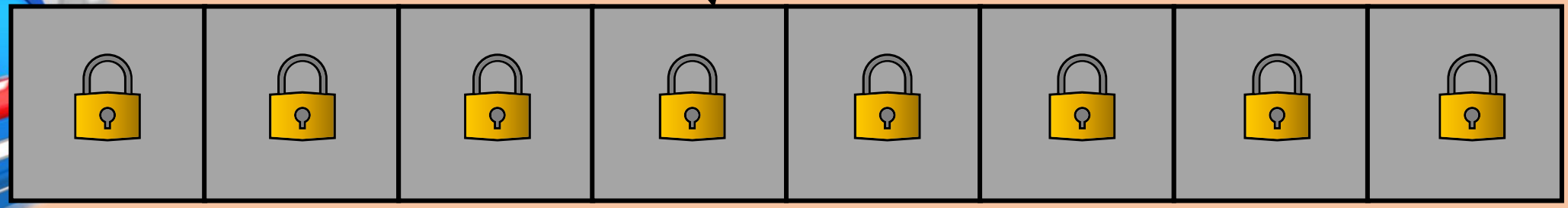
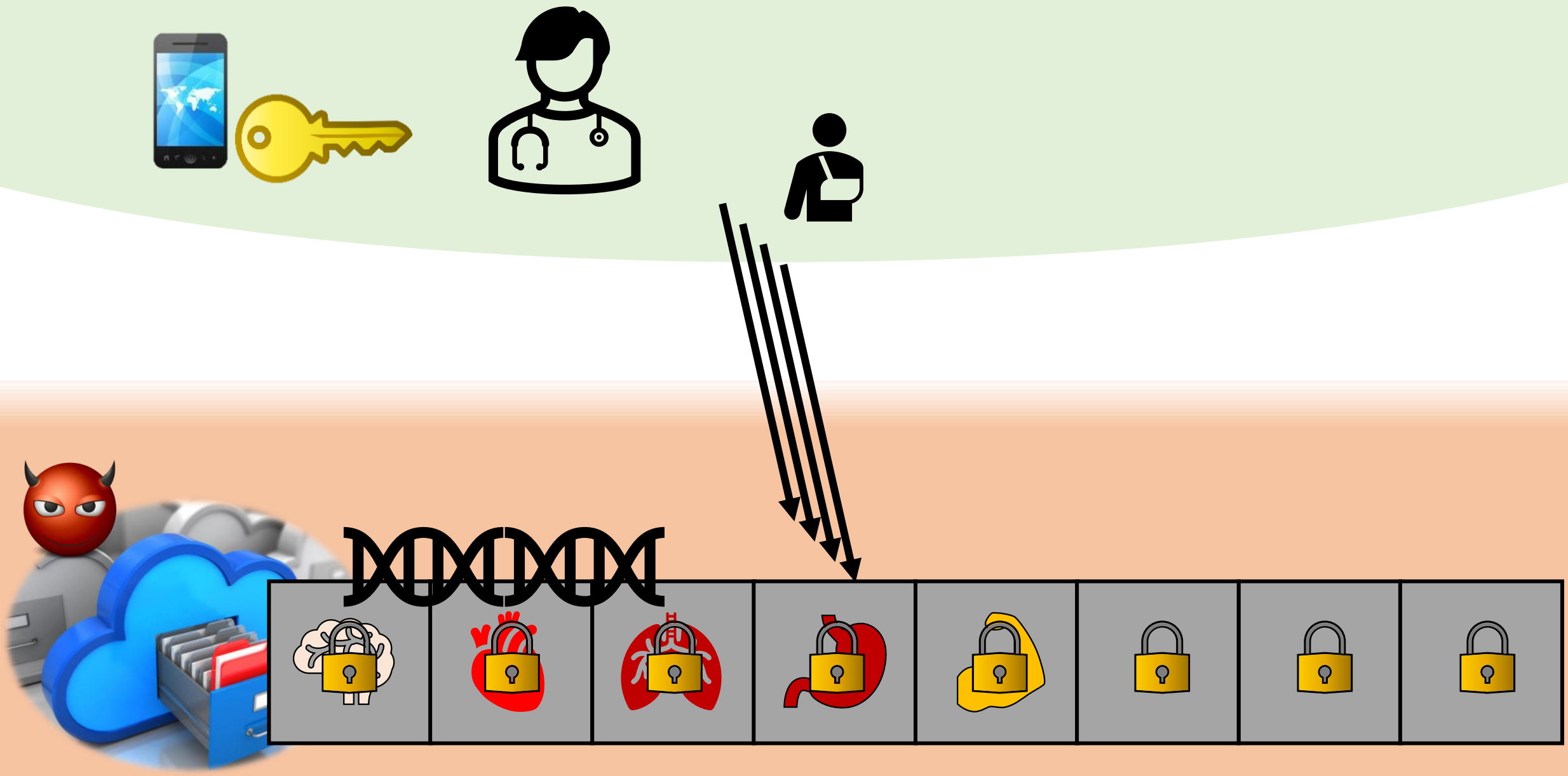
Joint work with

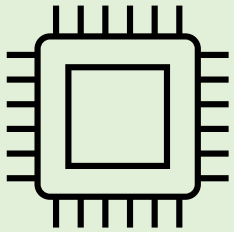
Gilad Asharov, Ilan Komargodski, Enoch Peserico, Elaine Shi

Access pattern leaks data

Frequency,
Correlation





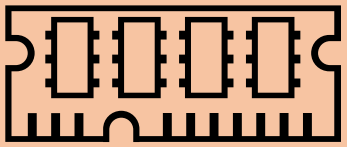


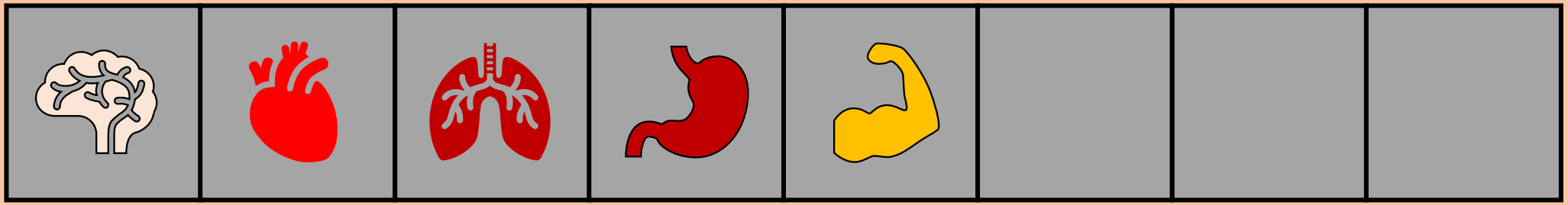
Oblivious algorithms:
Locations “indep of”
(secret) data

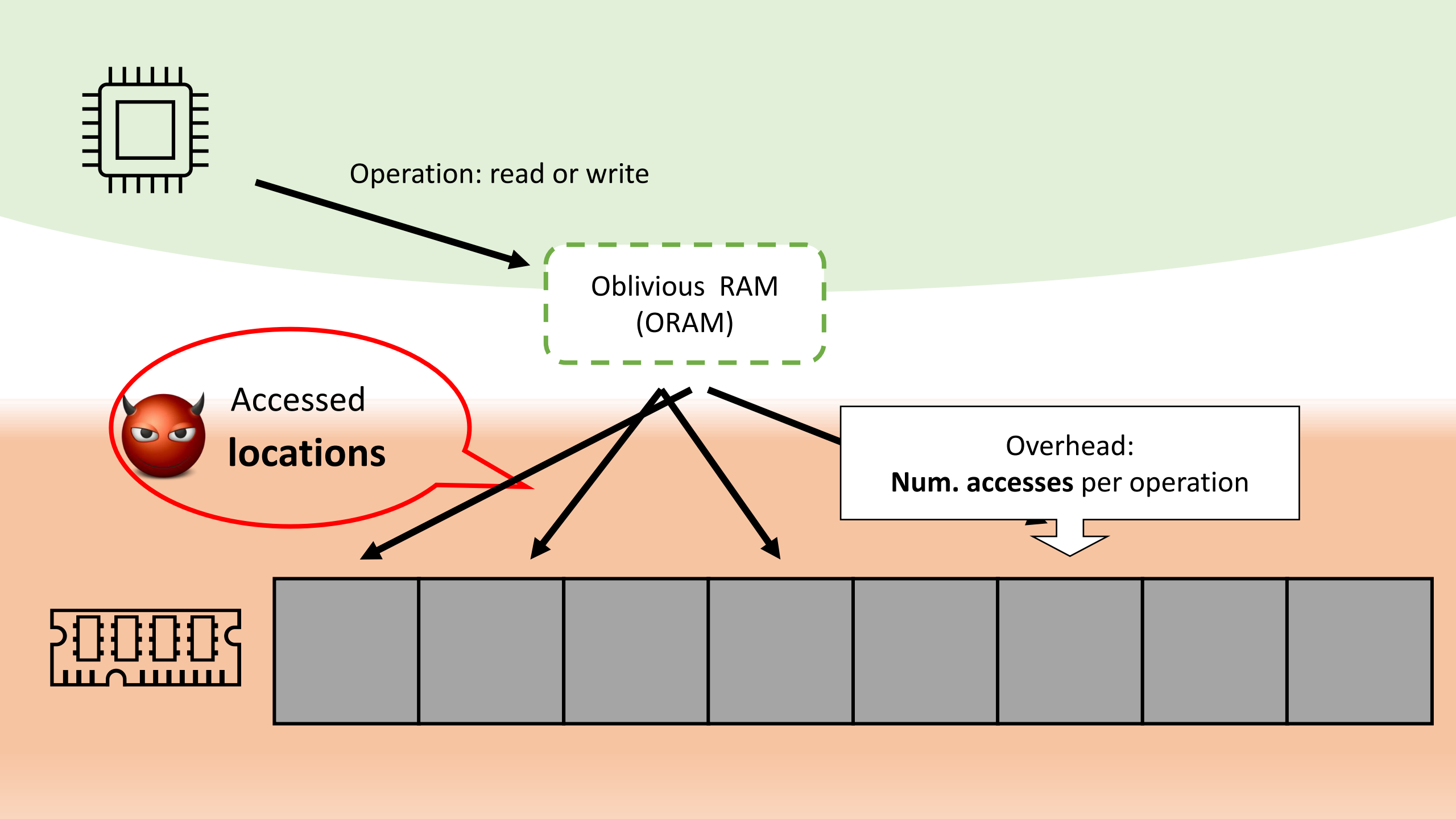
[Goldreich-Ostrovsky87,96]

Accessed
locations

Infinite algos / data structs?
Yes, **Oblivious RAM**

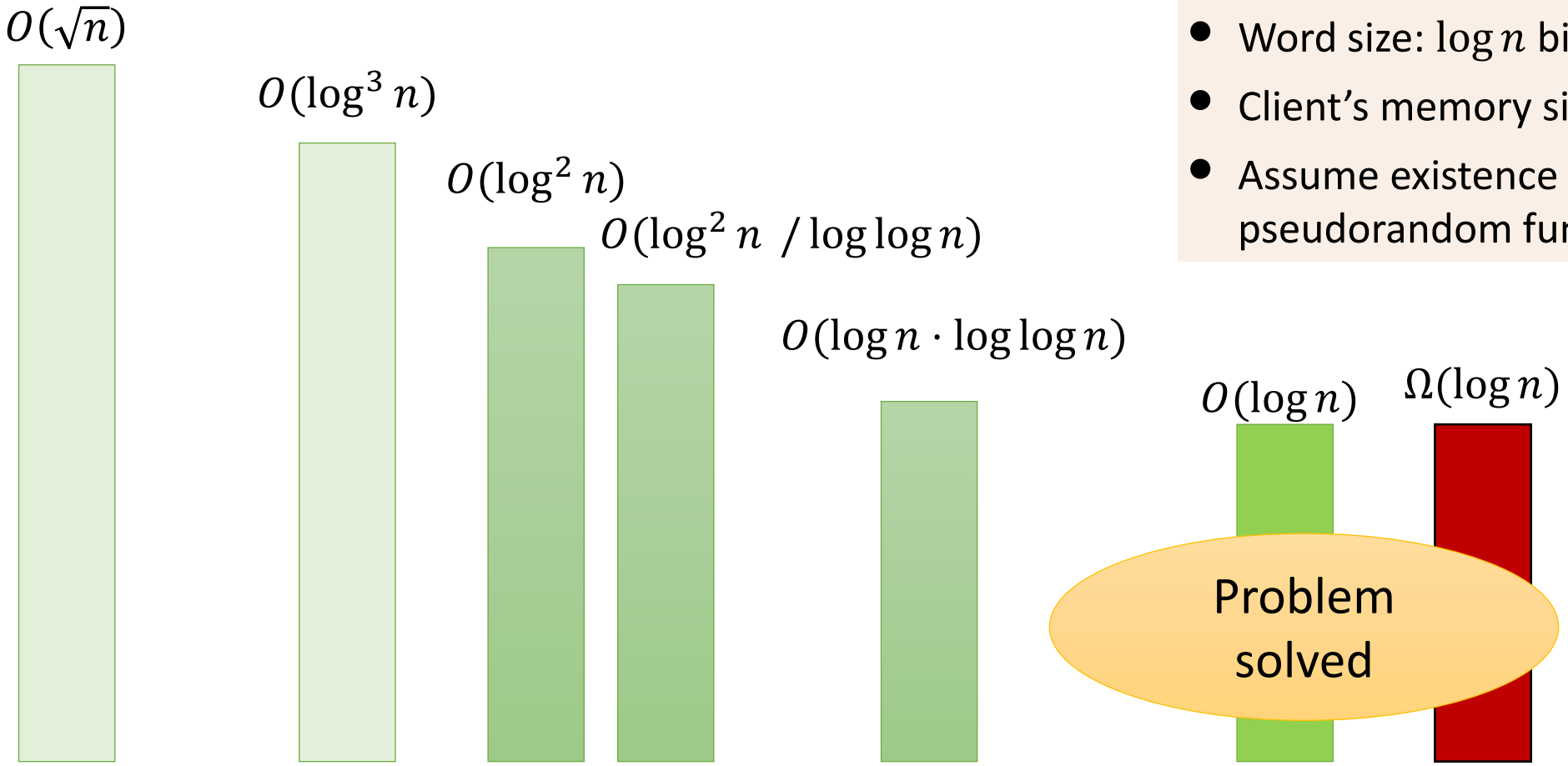





Operation: read or write
Oblivious RAM
(ORAM)
Accessed
locations
Overhead:
Num. accesses per operation

ORAM Overhead, Standard Setting

- n: size of the memory
- Word size: $\log n$ bits
- Client's memory size $O(1)$ words
- Assume existence of cryptographic pseudorandom function



[G87]

[GO96] [DMN11] [SCSL11]

[GM11] [SvDSF+13] [WCS15]

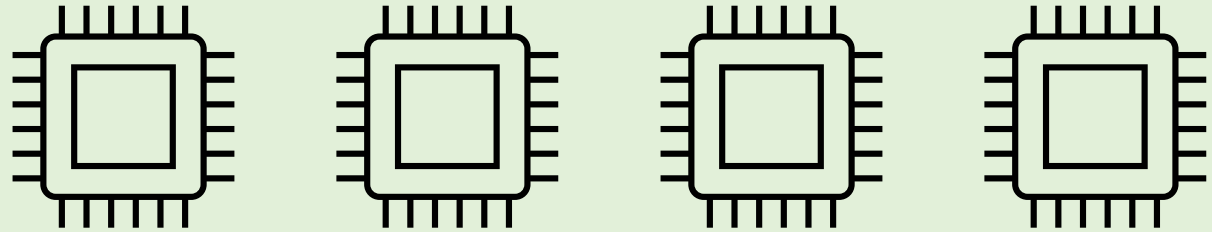
[KLO12]

[PPRY18]

[AK**L**NPS20] "OptORAMa"

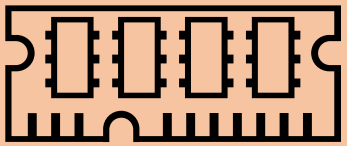
[GO96] [LN18]

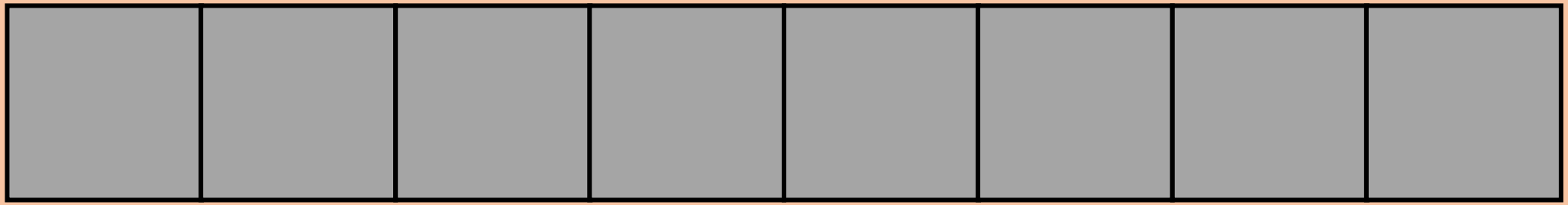
[Boyle-Chung-Pass16]

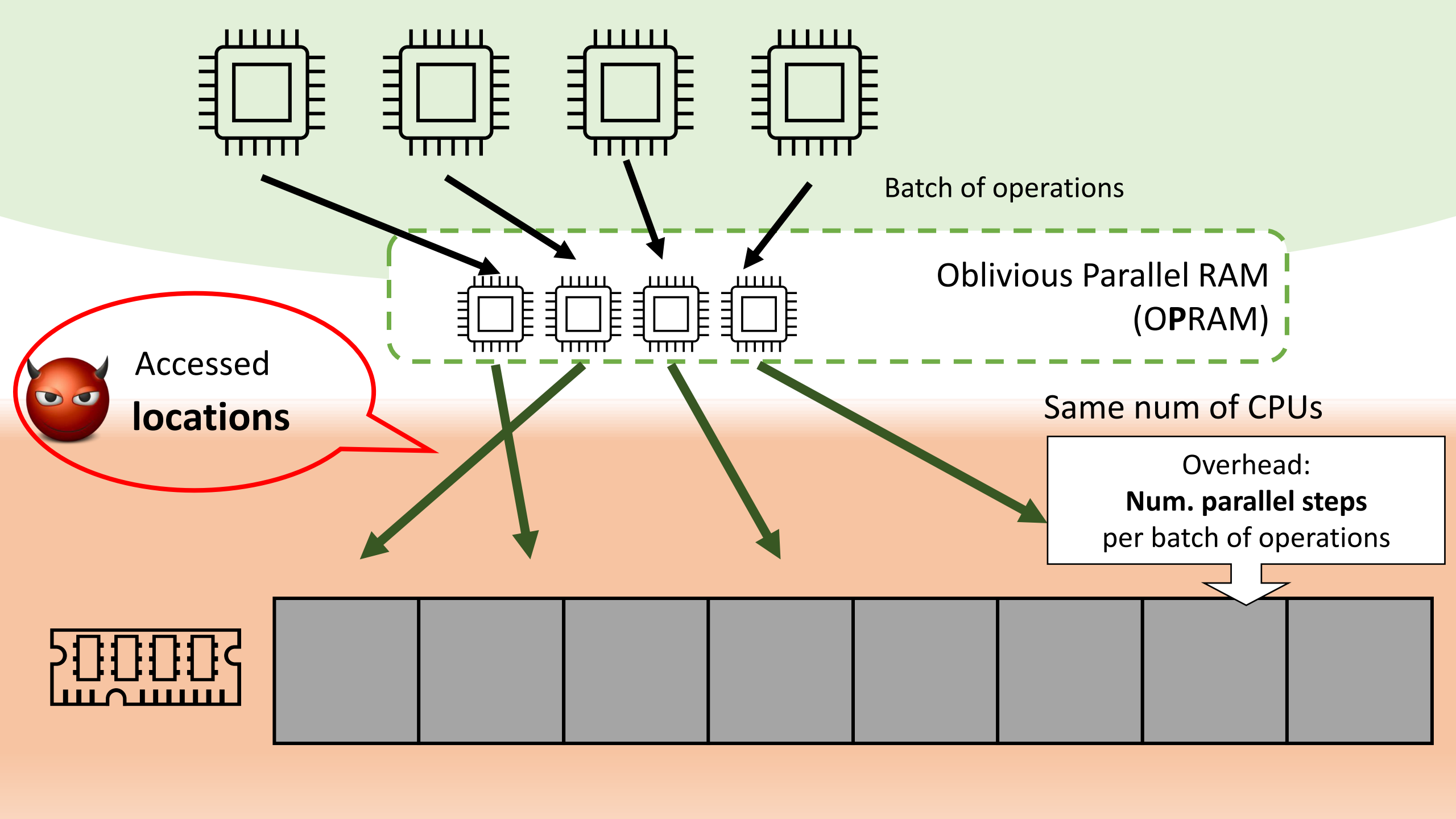


Parallel RAM?
Oblivious **P**RAM

Accessed
locations

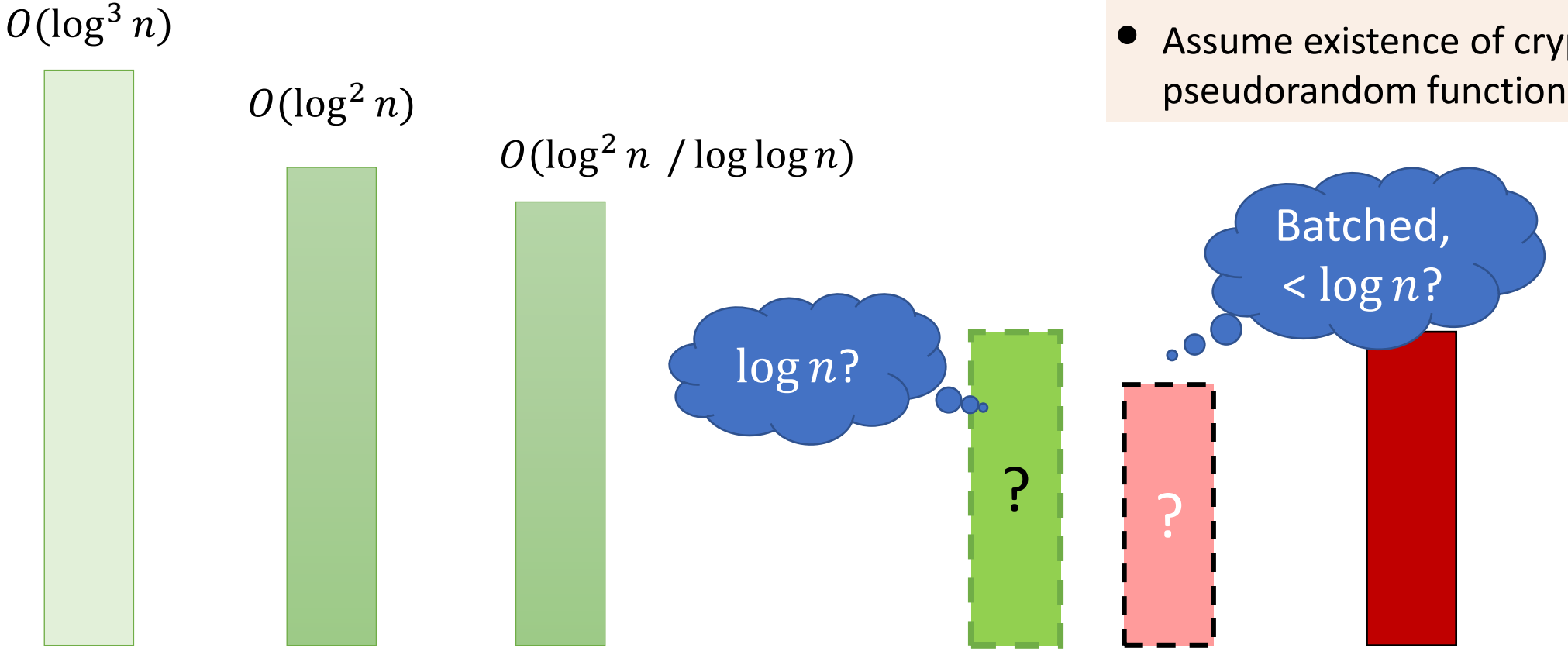





Batch of operations
Oblivious Parallel RAM
(OPRAM)
Accessed
locations
Same num of CPUs
Overhead:
Num. parallel steps
per batch of operations

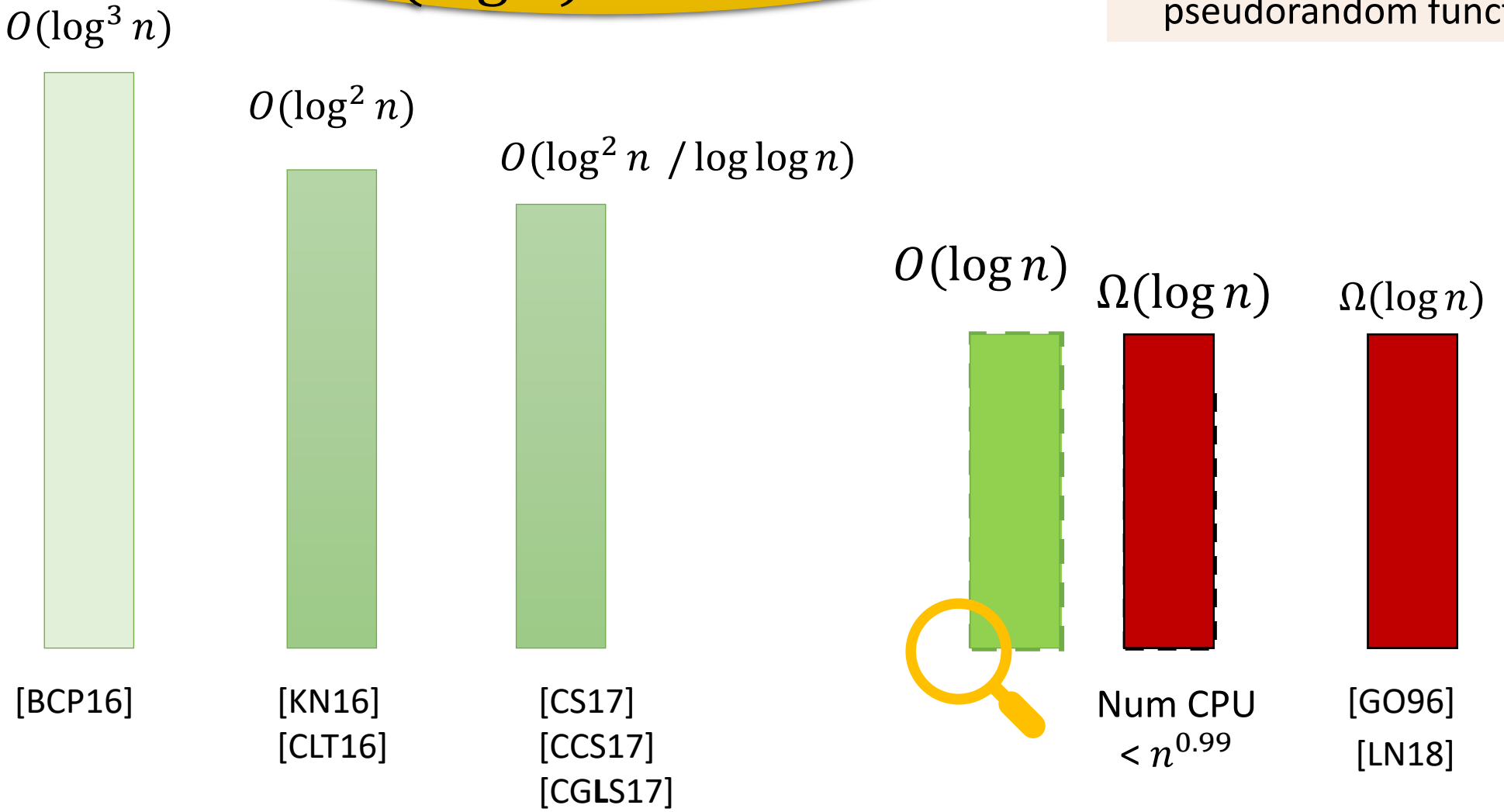
Main question: OPRAM overhead?

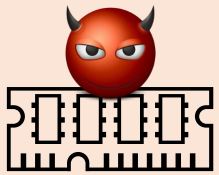
- n: size of the memory
- Word size: $\log n$ bits
- Client's memory size $O(1)$ words
- Assume existence of cryptographic pseudorandom function



Main result: Optimal OPRAM, $\Theta(\log n)$ overhead

- n: size of the memory
- Word size: $\log n$ bits
- Client's memory size $O(1)$ words
- Assume existence of cryptographic pseudorandom function





Hierarchical Framework

[Goldreich-Ostrovsky '87,96]

Put ● into Level 0

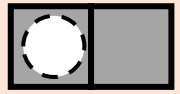
Rebuild (hash)
level i per 2^i operations

Level i, capacity 2^i

0

$H_1(5)$

1, H_1

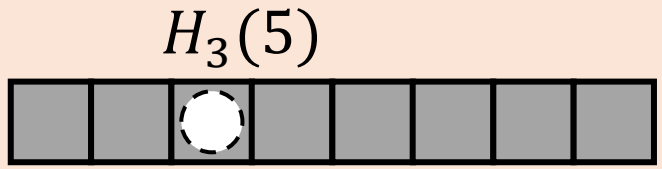


$H_2(5)$

2, H_2

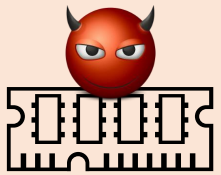
$H_3(5)$

3, H_3



Oblivious Hash Table:

- "Build": store 2^i balls
- "Lookup" for key k: find and output k and associated ball
- Build + Lookup is oblivious: not reveal balls even adversary sees accesses



Optimal ORAM (sequential)

[PPRY18] "PanORAMa" | [AKLNPS20] "OptORAMa"

Oblivious Hash Table
- Build: linear time
- Lookup: const time

Level i, capacity 2^i

0

$H_1(5)$

1, H_1

Rebuild (hash)
level i per 2^i requests

$H_2(5)$

2, H_2

$H_3(5)$

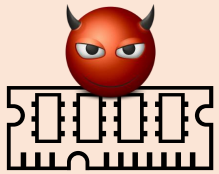
3, H_3

Linear time building blocks

"Tight compaction":
Oblivious sorting balls using 1-bit keys

"Intersperse":
Mixing 2 lists uniformly at random

"Toss balls into bins,
and then report loads of bins"



Oblivious Parallel RAM

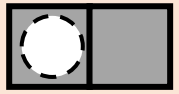
Also need:
$O(\log n)$ parallel time

Level i, capacity 2^i

0

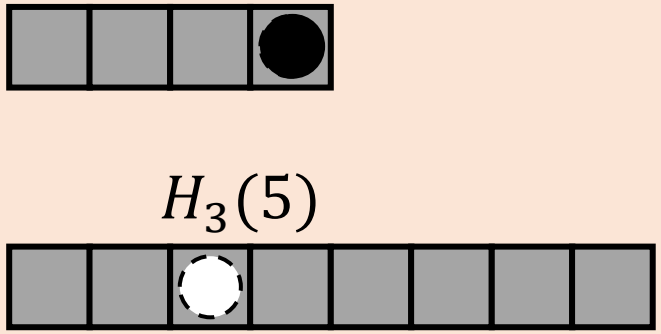
$H_1(5)$

1, H_1



$H_2(5)$

2, H_2



$H_3(5)$

3, H_3

Oblivious Hash Table
- Build: linear work
- Lookup: const work

Linear work building blocks

Yes! [AKLPS20]*

"Tight compaction":
Oblivious sorting balls using 1-bit keys

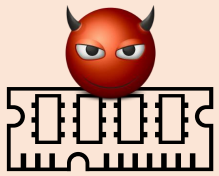
Yes! [This work]

"Intersperse":
Mixing 2 lists uniformly at random

Yes! [This work]

"Toss balls into bins,
and then report loads of bins"

*Related: "optimal sorting circuits" (Wednesday 4:30, Session 12B)



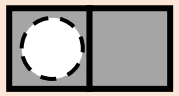
Oblivious Parallel RAM

Level i, capacity 2^i

0

$H_1(5)$

1, H_1



$H_2(5)$

2, H_2

$H_3(5)$

3, H_3

Building blocks take:
Linear work
$O(\log n)$ parallel time

$\Rightarrow$

OPRAM:
$\sim \log^2 n$ overhead

$\log n$ levels,
each takes
$\log n$ parallel time

Pipeline levels [This work]

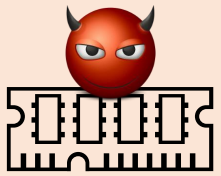
$\Rightarrow$

OPRAM:
$\log n \cdot poly \log\log n$
overhead

Main Technical Challenge

Oblivious Hash Table

- Build: linear work,
  $\log n \cdot poly \log\log n$ parallel time
- Lookup: const work

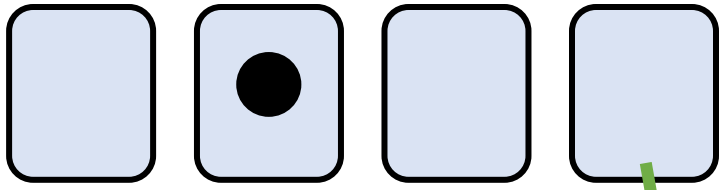


Oblivious Hash Table

[PPRY18] [AKLNPS20]

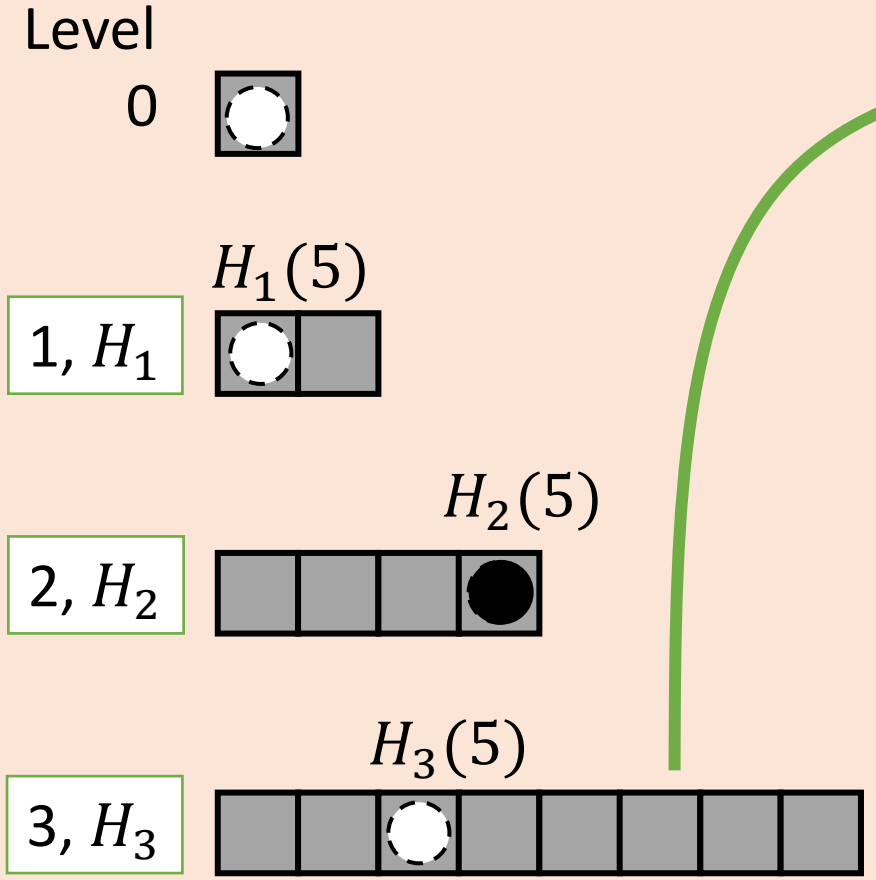
1. “Balls into bins” hashing

Choose the bin by H(addr)



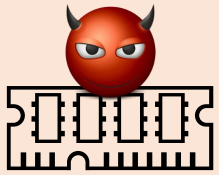
Poly log bin-size➔ negligible overflow prob.

2. “Cuckoo” hashing within each bin



✕ Build cuckoo hashing: linear work, $O(\log n \cdot poly \log\log n)$ parallel time

Cuckoo: Const-time lookup



Level

0

$H_1(5)$

1, H_1

$H_2(5)$

2, H_2

$H_3(5)$

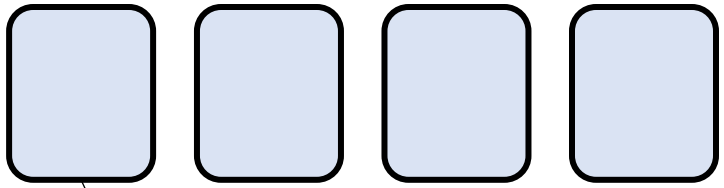
3, H_3

Wanted Hash Table

- Build: linear work, $\log n$ parallel time
- Lookup: const work

"Balls into bins" hashing

Choose the bin by addr



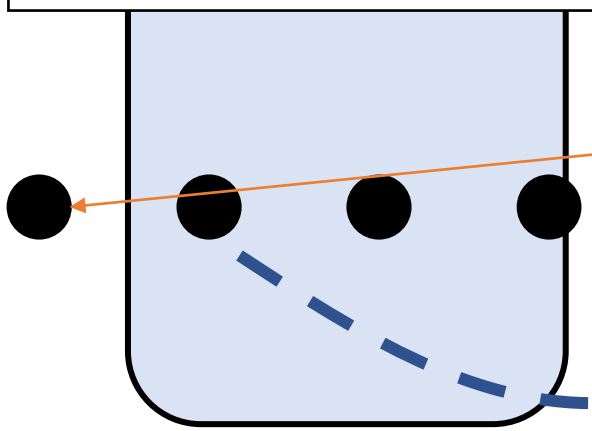
"Short Hash Table"

A. Random shuffled input

B. "Amortized" over many bins
Build many bins
Lookup $\log n$ bins

- Word size: $\log n$ bits
- Ball: (key, value) pair, $\log n$ bits
- Bin size: s = poly log n words
- Num balls: $O(s)$

A. Random shuffled input



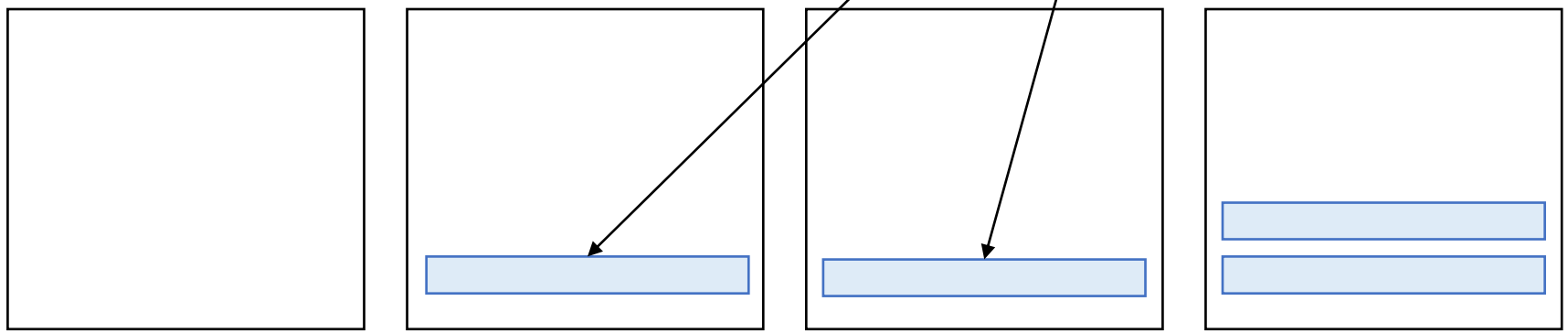
micro ptr, micro key,	micro bin
micro ptr, micro key,	micro bin
…	micro bin
…	micro bin

1 micro record per ball,
$O(\log \log n)$ bits

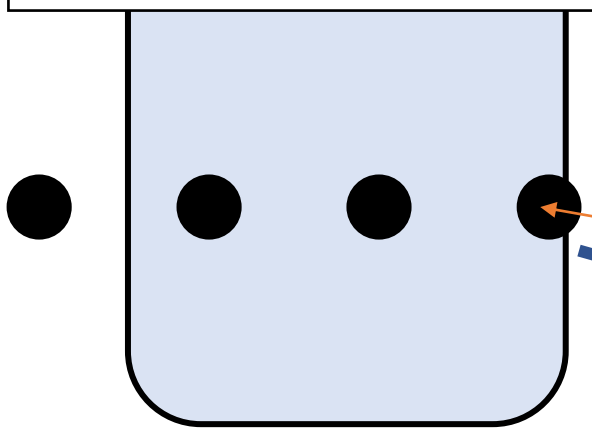
(micro key, micro bin) := H(k)

s Micro bins, each $O(1)$ words

➔ Each stores $O(\log n / \log \log \mathrm{n})$ micro records



- Word size: $\log n$ bits
- Ball: (key, value) pair, $\log n$ bits
- Bin size: s = poly log n words
- Num balls: $O(s)$

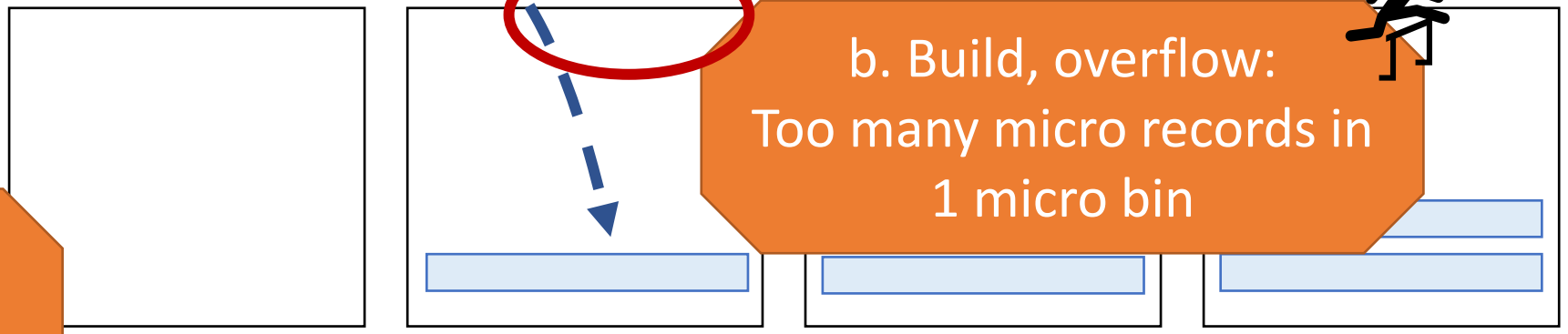


a. Build: Need time to pick **unique** micro keys

micro ptr | micro key, | micro bin

1 micro record per ball, $O(\log \log n)$ bits

(micro key, micro bin) := H(k)

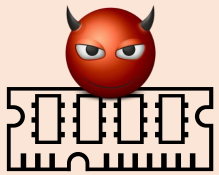


b. Build, overflow: Too many micro records in 1 micro bin

c. Lookup, false positive: key' not in this index hits same micro key

s Micro bins, each $O(1)$ words

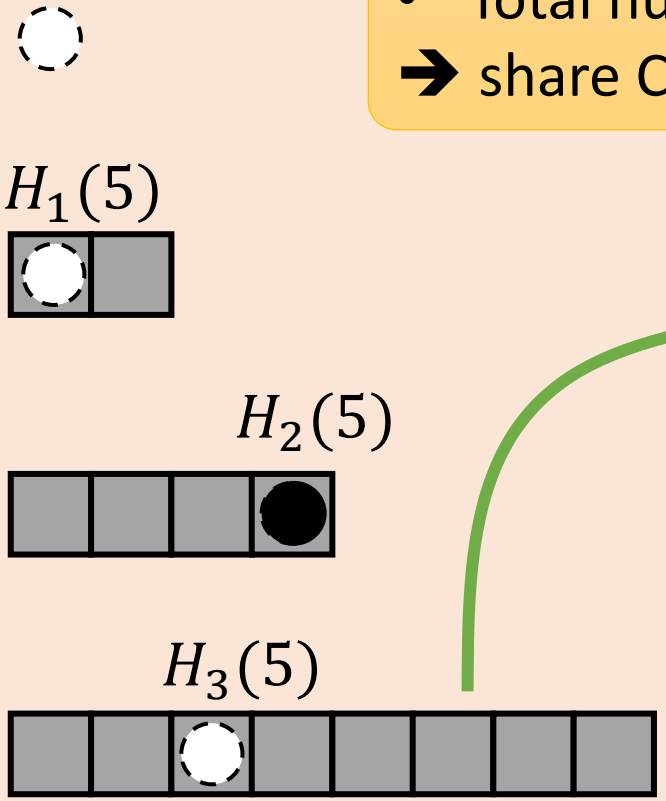
➔ Each stores $O(\log n / \log \log n)$ micro records

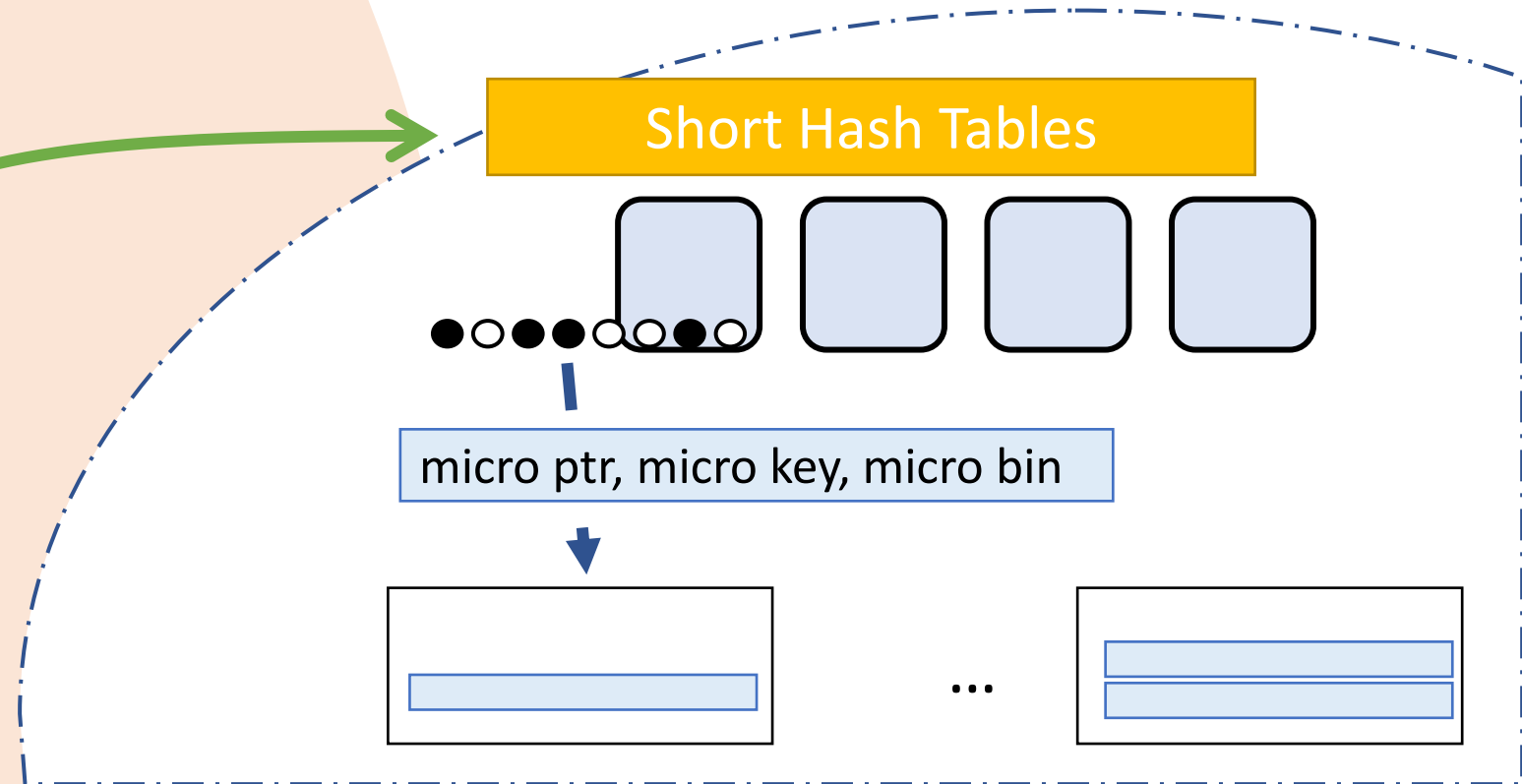


B. “Amortized” over many bins

- Build:
  $> poly \log n$ instances of ShortHTs (in parallel)
- Lookup:
  $\log n$ instances of ShortHTs (sequential)
- Total number of instances $< n$

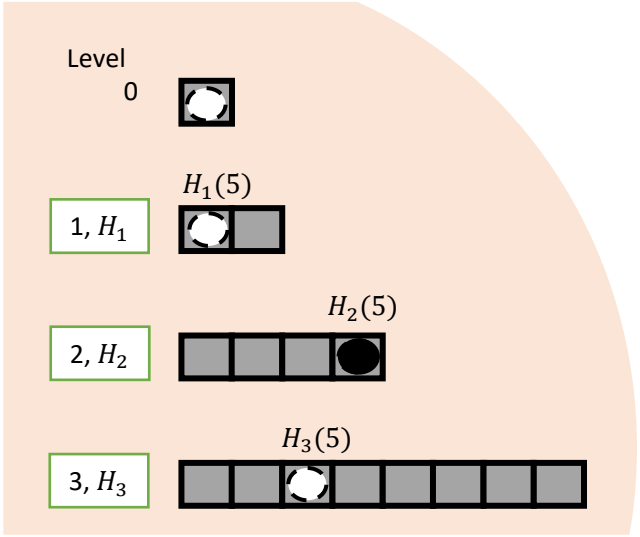
➔ share CPUs, overflow/false positive space

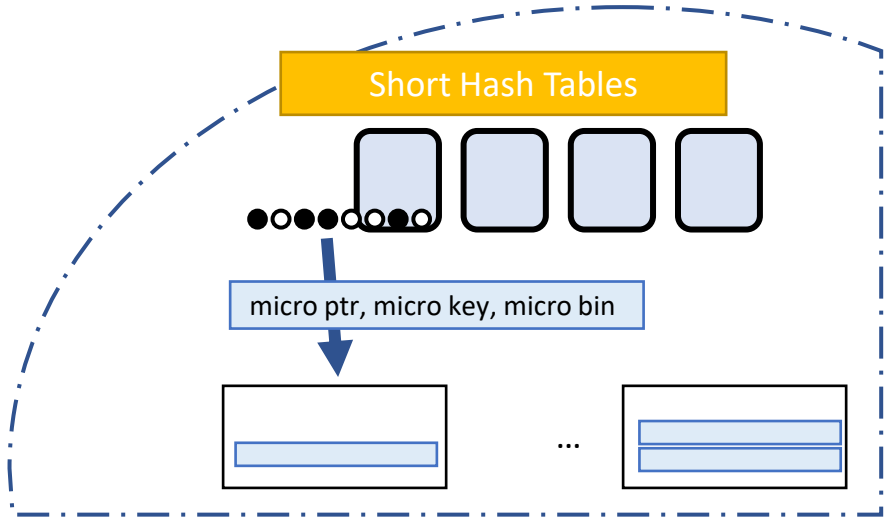




Omitted Details (need oblivious and parallel):

- Pipeline $\log n$ levels of hash tables into Oblivious PRAM
- Building blocks
- Lookup Short Hash Table concurrently





- Extend lower bounds of [GO96] and [LN18] to parallel

Thank you!